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AUTHOR Walberg, Herbert J.
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INSTITUTION Illinois Univ., Chicago. Chicago Circle Campus.
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ABSTRACT

To solve problems of causality and measurement in educational research, this paper combines seven variables into a proposed model of educational productivity on achievement tests. The authors first review psychological models of educational production that relate learning to aptitude and environment, and note that these models do not allow for interaction between the aptitude and environment variables. The authors then describe productivity models in economics and demonstrate that such models are explicit and testable and do allow for interaction among variables. An educational model emulating the economics models is proposed to explain academic achievement on standardized tests. In this model of educational productivity, an equation combines the variables of student ability and motivation, instructional quantity and quality, home and classroom environments, and age. According to the authors, the interaction among the model's variables may explain a number of problems in educational research, including the lack of demonstrable results from instructional innovation or compensatory education, the weak correlations between achievement and instructional time, and the confusion in findings relating aptitude to teaching methods. The paper's final section stresses the need to test the proposed model on long-term empirical data. (RW)

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A Psychological Theory of Educational Productivity

Herbert J. Walberg

University of Illinois at Chicago Circle

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A Psychological Theory of Educational Productivity[1]

Herbert J. Walberg

University of Illinois at Chicago Circle

Two perennial questions in education are: "What are the ends of education?" and "Do the educational means, that is, the manipulations of the environment, justify the ends?" These philosophical questions concern values, morality, and ethics. Modern psychology and social science raise additional points: "Can the ends and means of education be measured?" and "Do the presumed means in fact cause the ends? If so, to what extent or with what degree of productivity?"

Although all these questions are important and incompletely settled, the last two concerning causality and productivity are of major concern here. On this question, education follows medicine, agriculture, and engineering: the physician wants to know the cure rates of groups of patients subjected to different drugs or treatments and possibly a no-treatment control condition; the agronomist estimates the yield responses to various amounts of fertilizer and rainfall; and process engineers find the effect of control settings on output. After these estimates of causal relation are made, policy can be specified within an explicit, objective, rational framework. The feasible treatment or combination of treatments that maximizes outputs can be recommended. Or, if costs of the treatments and the level of the outcome are known, the least expensive feasible treatment or combination can be identified. Judging from the accumulated research, however, such causal relations and most efficient policies are not easily identified in education.

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To determine educational policy and practice requires not only the valuing of certain ends but the assumption that the means employed indeed cause the ends in question. Experiments with random assignments to treatments are ideally suited to probe or assess causality, but Gilbert, Light, and Mosteller (1976) in a recent review were able to find what they considered to be two well-executed field experiments in education. Although the stringency of their criteria might be relaxed to include more experiments, it seems that experiments are difficult to implement in natural settings of education. Two alternatives, quasi-experiments and path analysis, also present difficulties in elucidating causation. Quasi-experiments in practice usually only control for one or two rival causes rather than a reasonably complete list of the known correlates of educational achievement; and path analysis or structural equations as employed in behavioral research usually beg causal questions by assuming rather than probing their answers. Thus educational research workers who wish to base policy on creditable causality confront the dilemma of choosing among experiments that are difficult to implement and quasi- experiments and path assumptions that are difficult to defend. A promising way to resolve this dilemma, it is argued below, is to include in both experimental and non-experimental research the chief known correlates of educational achievement: age or developmental level; ability, including prior achievement; the social environment of learning or allocated for learning; and the home environment.

Psychological Production Functions

Many psychological theories can be subsumed by Lewin's (1963) general formulation of behavior as a function of personality and environment

$$B = f(P,E)$$

educational researchers have implicitly or explicitly adopted a similar general equation:

$$L = f(A,T)$$

in which learning is a function of individual aptitude and instructional treatment. Some have preferred to include

$$L = f(A,T,E)$$

environment in the equation since in natural settings of education, non-manipulated factors often seem to carry more weight in producing learning than deliberate instruction (Walberg, 1976); and since observational research (without treatment randomization) requires all determining factors in the equation for causal inference.

Among the problems of these general equations in educational and psychological research are: agreeing on a list of the parts of each of the constructs; operationally defining and measuring each part; stating the rules for adding up the measurements for an overall estimate of each construct, if required; specifying the form of the equation; and estimating its coefficients. The discussion here touches all of these problems but is mainly concerned with specifying the form of the equation and applying it to the explanation of several educational production problems. Operational measurement issues are treated in the references cited.

For simplifying the discussion in this subsection, consider that instructional treatment is a part of environment in the last equation so that learning may be considered a function of individual and environmental

factors. For further simplification, consider that the parts of each have been optimally defined, measured, and summed to provide overall indexes of the two constructs

$$L = f(A,E)$$

The usual form of the regression (of which t and analysis of variance and covariance are special cases) for estimating learning is

$$L = a + bA + cE$$

where a is a constant and b and c are coefficients or regression weights for the aptitude and environment terms. In normal science, the estimated coefficients would explain, account for, or predict the amount of learning. For example, assuming linearity and causality, weight c would estimate how much increased learning would take place by increasing the environmental factor by one unit. Such equations are usually unreplicated because different combinations of constructs and measures are used in each study, and the estimated weights are often unreported.

A possible problem of the equation itself, however, is that it does not allow for the possible "interaction" of aptitude and environment (Lindquist, 1951; Cronbach, 1957): for example, a student with high visual aptitude may benefit more than others from instruction making use of pictures. Such interactions may be tested by adding an arithmetic product term (not to be confused with educational or economic product) AE to the regression

$$L = a + bA + cE + dAE$$

Although such interactions seem educationally sensible, they also are generally found weak and difficult to replicate (Bracht, 1970; Walberg, 1976) not only because of the variation in constructs and measures but also because the high correlations of aptitude and environment with their product result in large errors in estimating the weights. Moreover, the extra product term is scientifically and practically unparsimonious unless it is clearly needed. A different kind of interaction seems required.

An Economic Production Function

The notion of interactions of aptitudes and environments may yet bear fruit; part of the problem may lie in the form of the equation. Econometric production functions incorporate interactions parsimoniously and also suggest other analogs worth attempting to apply in education.

Econometricians often assume that output production, nationally or at the factory level, is a function of capital and labor (subsuming land under capital for simplicity of illustration)

$$O = f(K,L)$$

This simple specification often serves to account extremely well for economic processes that are known to be highly complex. Samuelson (1962) showed "how we can sometimes predict exactly how certain quite complicated heterogeneous capital models will behave by treating them as if they had come from a simple generating production function" (p. 194). Solow (1956), in a discussion of economic growth, commented: "The art of successful theorizing is to make the inevitable simplifying assumptions in such a way that the final results are not very sensitive" (p. 65).

The most celebrated form of the production function in economics is attributed to Cobb and Douglas (1928) although Wicksell and Wicksteed put it forward before the turn of the century (Jones, 1976) and were clearly cited by Cobb and Douglas. A clear exposition and Douglas's replication of its coefficients in various countries and time periods led to its widespread use. Other more complicated production functions by Arrow, Leontief, and others (discussed by Jones) may eventually prove superior but they presently lack the long record of successful empirical verification of the Cobb-Douglas form and are beyond the scope of the present treatment (see Bosworth, 1976 Jones, 19[illegible] for recent comparisons of functions applied to various industries and firms).

The classical Cobb-Douglas function may be written

$$O = aK^bL^c$$

where a is a constant and b and c are the coefficients for capital and labor. The coefficients may be conveniently estimated in linear regression by simply taking logrithms of all the variables.

$$\ln O = \ln a + b \ln k + c \ln L$$

Following Wonnacott and Wonnacott (1970, pp. 91-98) and Jones (1976, pp. 25-35) the function incorporates several valuable properties:

1) Positive marginal returns: The marginal products of capital and labor are both positive, that is, adding either more labor, or more capital increases output (at least up to a point). For example, adding more or better machines, or more workers or overtime, [illegible] measured by amounts or costs, increases output. Thus b and c are both positive.

2) Diminishing marginal returns: Successive equal increments in either capital or labor, without increasing the other factor, results in smaller and smaller increments in output (although such diminishing returns may not set in immediately). For example, doubling the work force of a farm, factory, or nation will less than double output if capital (including land) remains constant. Thus b and c are both less than one.

3) No input, no output: If either no capital or no labor are used, no production can ensue. A value of zero for either K or L or both results in a value of zero for output in the equation since any number multiplied by zero equals zero.

4) Constant returns to function: Increasing both capital and labor by any factor or percent increases output by the same factor or percent. For example, doubling both K and L doubles O. The sum b and c is assumed and often is found to equal approximately one.

5) Changing substitutability between labor and capital: Additional labor can substitute or trade off for diminished capital, but more and more labor must be added to compensate for each unit reduction in capital; and vice versa. Thus the function is multiplicative or interactive rather than additive. The productivity of labor, or O/L, output per hour or worker increases with increasing capital per worker, that is, the capital-labor ratio, K/L, but decreases with increases in labor alone (the diminishing returns case).

6) A measure of effect: Taking logrithms of output means that the regression weights are estimated percentage increases in output with a one unit increase in the respective independent variable. Taking logrithms of the independent variables means that they are also being expressed as percentages. Thus the original Cobb-Douglass (1928, p. 151) time-series

regression estimates of the coefficients

$$O = 1.01\ L^{.75}K^{.25}$$

for the U.S. economy 1899 through 1922 mean that .75 and .25 percent increases in output are associated respectively with one percent increases in labor and capital (R = .97).

7) Developmental effects: Although the production function proved useful in fitting both longitudinal data for single nations, industries, and factories and also cross-sectional comparisons of samples of these units, Solow, Samuelson, Hicks, Kuznets, and others, point out that the increasing efficiency of industry beyond that which can be explained by increases in capital and labor inputs suggests that a third factor must be added to the equation, technological progress brought about by education, science, engineering, and management (Meade, 1961). While labor and capital can be calculated fairly conveniently from national and corporate statistics, technology cannot; so time is used as a proxy in the general formula

$$P = f(L,K,t)$$

In the neo-classical Cobb-Douglas form time may be entered as an exponent t

$$P = aK^bL^ce^t$$

where e is the base of the natural logrithm 2.71....

Points 1 through 3 are illustrated in Figure 1. Increases in capital alone or labor alone increase output but at a diminishing rate. When a

factor is zero, no output ensues.

Similarly Figure 2 illustrates points 4 and 5. Increasing both factors by the same amount produces the same amount of gain in output from Λ to Λ^1. One factor can substitute for the other, but at decreasing returns to the factor and increasing costs.

Figure 3 illustrates points 6 and 7. The purpose of the production function is to explain change in output per unit time, beyond that attributable to natural technological growth with time, as a function of changes in amounts of capital and labor.

Production in Education

Economists are blessed with such a strong model of production. Even if it can be shown to be clearly wrong, it is explicit and testable. Given only three or four variables that are reasonably well defined and measured, and an explicit form of the equation with only a few coefficients, the estimates of the function can be repeated and possibly replicated. And the results and forecasts, as a matter of academic interest or policy decisions, can be expressed in as universal and easily understood percentage changes, notwithstanding differences in metrics and units.

Perhaps Lewin's formulation of behavior as a function of personality and environment comes as close to the general production equation as any other in psychology. If transitional processes such as perception are set aside, Lewin's split of forces within and outside the individual seems reasonable. But psychology has reached very little consensus on the list of parts of each construct, their definition, measurement, and scoring, and the specific, explicit form of the equation.

If the problem is narrowed, however, from behavior in general to academic achievement on standard tests, then a preliminary list of the production factors may be drawn up on the basis of their reasonable consistency in predicting variance in achievement (Figure 4). Such estimates of variance accounted for (see Walberg, 1971, 1976) undoubtedly have large margins of error depending on: sampling, unreliability of the measures; the level of aggregation, for example, indiviudal, class, or school; the lack of variance of some factors which prevents the detection of their covariance with achievement; the limited range of achievement tests themselves; and possibility of non-linear covariations. Nevertheless, these estimates may serve provisionally to prune the many variables that might enter equations.

There are plausible theoretical rationales and empirical evidence on the significance of the relations (in Figure 4) in the published literature from which the estimates have been drawn (Berliner, 1978; Bloom, 1976; Rosenshine, 1978; Walberg, 1971, 1976; and Walberg and Marjoribanks, 1976). A problem within the research in this literature is that the variables are often studied in isolation rather than in concert, because of the division of psychology and sociology, sub-divisions within these fields, and specialized individual research interests.

It is possible to prune the list further, and the variables in the left-hand column may be tentatively deleted. First, heredity cannot be directly measured; it and the other variables in the column seem reasonably well mediated by those in the second column. For example, the quality and quantity of adult supervision in the home environment are much more closely related to achievement and achievement gain than is an index of parental education

and income and number of siblings in the home (Walberg and Marjoribanks, 1976). Similarly it seems that teacher age, sex, experience, and even personality are superfluous once the teacher's action or the environmental press on the student is taken into account (Rosenshine, 1978).

Rearranging the production factors, specifying some in measurable policy detail and adding age yields Figure 5. Two or three factors would be more parsimonious but seven seem required because of their plausible and empirical correlations with achievement. Indeed, in some applications it may be desirable to disaggregate the variables more finely; research on instructional quality may include each of the 5 to 15 teaching variables thought to promote student achievement (Rosenshine, 1978) rather than an overall weighted index.

There is still considerable redundancy or colinearity among the seven independent variables. For example, measures of cognitive aptitude and home environment each account for about 60 percent of the variance of achievement, but together they account for about 80 percent. (Note that these percentages of variance and those in Figure 4 are the squares of the correlations between the factors and achievement not the more desirable percentage forecasts from multiple logrithmetic regression such as in the Cobb-Douglas function.) Similarly cognitive aptitude and student perceptions of the class environment account for about 90 percent (Walberg, 1971, 1976). Generally a student high on any variable, except age, is more likely to be high on the other variables: to them that hath shall be given. But a planned sample should produce enough exceptions to hold down the correlations among the independent variables sufficiently relative to the multiple correlation. Thus, despite redundancy, variables in a complete equation are expected to carry significant weight in achievement regressions; that is, they should each explain unique variance in achievement, provided they are resonably well measured and vary

over a moderate range.

The best equation, in the writer's opinion, the one that would allow the best basis of causal inference and estimates of effects would include all seven variables in the following Cobb-Douglas form

$$Ach = a(Abl)^b(Mot)^c(Qul)^d(Qun)^f(Cls)^g(Hom)^h(Age)^i$$

The coefficients for the equation are conveniently estimable in a linear regression by taking logrithms

$$\ln (Ach) = \ln a + b \ln (Abl) + c \ln (Mot) \ldots$$

By analogy to the earlier points, the following provisional hypotheses can be put forth:

1. Increasing any production factor (as distinct from the age factor, a proxy for growth processes) increases achievement; coefficients b through h are positive (see, however, the note to Figure 1).

2. Increasing any factor while holding the others fixed produces diminishing marginal returns; b through h are each less than one (Figure 6).

3. A direct extension of the economic production function is that any factor equal to zero results in zero achievement. Unlike capital and labor, however, the educational production factors may not have validly measureable zero points. Thus, it is more reasonable to hypothesize that when any factor is near minimum, it is unlikely that achievement will be high, unless the other factors are near their maximum levels.

4. Increasing all the factors by a given amount raises achievement by the same amount. The sum of coefficients b through h approximates one.

5. The factors substitute or trade off for one another but at diminishing rates of return.

6. The coefficients b through h are estimates of the percentage increase in achievement associated, or, if causality is imputed, determined by a one percent increase in the corresponding factor. It would be possible to plot a profiles of the factors for diagnosis and forecasting of achievement production for each student. Those achieving productively will have high and flat profiles. Unproductive students will have low, uneven profiles; and raising the lowest factor would produce the greatest returns, although it may be costly, difficult, and beyond the control of educators.

7. Comparable to time which serves as a proxy for technological progress in the neo-classical function, age serves as an index of maturation (Piaget, 1954) or "spontaneous schooling" (Stephens, 1968) that seem to occur inexplicably, incidently, and unintentionally with increasing age apparently independently of the known production factors. Given a wide span of age and a wide range measure of achivement, the growth curve may turn out to be linear, exponential, or logistic. Presumably if the production factors were constant, the age term would generate the growth curve. Varying the other factors would result in accelerations and decelerations around it. It may be further hypothesized that steady application of the factors produces more growth than fits and starts.

The seven variables in the function are known to correlate with academic achivement (Berliner, 1978 Bloom, 1976; Rosenshine, 1978 Walberg, 1976; Walberg and Marjoribanks, 1976). It may seem educationally plausible to impute causality to them, although it would be more defensible to do so after they are analyzed in full concert or, one might say, in competition with each other

because one or more may be sufficiently redundant with the others to delete. Moreover, if true experiments cannot be done, with random manipulation of the factors, then the best possible hope for causal inference is to include all possible causes in a regression.

Some of the factors, however, may partially mediate each other: a supportive home environment, for example, may lead to better motivation and the perception of a productive social environment in the class; and able students may stimulate higher quality and quantity teaching. In samples that are nearly homogeneous in age such as those from a single grade level, age may be ignored. Thus, incomplete regressions may be of some value. Moreover, feedback mechanisms such as higher achievement producing better motivation and classroom perceptions should be entertained. Such possibilities can be investigated using advanced econometric analyses (Jones, 1976; Wonnacott and Wonnacott, 1970). Simultaneously-analyzed panal data, that which is collected on many individuals over multiple time periods, will be most valuable for detecting such complex mechanisms. If the simplifying assumptions of the function are reasonable, however, even incomplete, or partially defective data should generally and replicably fit the function better than the usual linear and multiplicative functions.

Applications of Production Theory

Educational Innovation

The theory of the education productivity as formulated above, appears capable of subsuming a number of educational models, principles, and empirical regularities. Before illustrating these applications, it may be of interest of return to the discouraging methodological problem mentioned in the opening section of this paper and to use the theory to explain the apparent lack of effects of educational innovation.

Many federal-and state-supported educational innovations that have been evaluated in the last few decades are attempts to change the quality of instruction or teaching to raise measured achievement. Since quality of instruction is only one of the six production factors and the other five may remain unchanged, diminishing marginal returns would ensue (see Figure 1); or the other factors may vary systematically. In either case or both, anyone of the following points may explain the apparent lack of dependency of outcome on the innovative instruction:

1. Volunteers who already have nearly maximized their teaching effectiveness-participate; therefore the increased returns (past point n in Figure 1 are too small to be detectable.

2. Untransformed linear terms are used to fit curved, logrithmic relations.

3. The sample is drawn from a single school district relatively homogeneous in quality, even after the innovation; therefore, the increment in achievement with respect to a small change in quality is undetectable.

4. The other five production factors are unmeasured and are not brought into the equation. Therefore, any point on the smooth curve (Figure 1) is given five large random shocks resulting in a scatter of points that make the true returns to quality undetectable.

5. Two large random shocks due to measurement error of achievement and quality of instruction to any data point obscure the relation of achievement and quality.

6. Only ability and quality are included in the analysis; quality adds only 3 percent to the 60 percent of the achievement variance explained by aptitude; the variance increment approach underestimates the true returns to teacher quality.

7. The innovation and control groups overlap substantially in teaching quality; a nominal two-group categorization rather than a continuous measure

of implemented quality is made.

8. The "non-innovating" control teachers compete with the innovators by assigning a greater quantity of time to instruction in the subject unit.

9. Students in the "non-innovating" control group compete by doing more homework.

10. The quality-of-teaching index contains a mix of elements, most of which previous research has not shown to be productive of achievement.

These points are overlapping and concern design, measurement, and specification problems that in various combinations are sufficient to vitiate the field-evaluation research. A more reasonable evaluation of innovations (and probe of the production theory) would include reasonably complete representation of the production factors, valid measures of each, sufficient variation in instructional quality (and the other factors); and logrithmic transforms in accordance with the hypothesized function to account for the diminishing marginal-returns curvature and the multiplicative substitution, trade-off, or compensation.

Quantity of Instruction

Time scheduled, allowed or assigned for a given instructional unit by the teacher or, better yet, the fraction of this time the student actually engages in learning the unit content is turning out to be moderately consistent though apparently weak correlate of academic achievement (Karweit, 1976; Rosenshine and Berliner, 1977). The usual problems of operational definition, measurement, and lack of factor variation may account for the small correlations; but the production function suggests that misspecification may also be a problem. Bringing more of the production factors into the equation, finding data in which time varies over a greater range, and taking logrithms may help

considerably. More specifically, the form of the production function suggests non-linear relations among time and the other factors that seem more conceptually defensible than the linear assumptions that pervade the research.

Consider the concepts of acceleration and enrichment as consequences of the presumed linear trade-off of time and aptitude (Figure 6a), where aptitude is motivation or cognitive ability or a weighted combination. For a given criterion amount of achievement or "mastery" in acceleration models of school learning it seems reasonable to say that, other factors being equal, moderate-aptitude students require relatively moderate amounts of time, and that the brighter students require correspondingly less, and the duller correspondingly more. Extrapolating such linear reasoning to extremes, however, produces the absurd conclusions that geniuses learn instantaneously and that the feeble-minded require just a little more time than the subnormal. Figure 6b shows a more plausible trade-off: aptitude and time compensate for one another, other things being equal, at diminishing rates of marginal returns; ever larger amounts of time, approaching a poorly-measured asymptote at infinity, are required to substitute for equal decrements of aptitude. Thus, to attain the highest performance levels will require at least a small amount of time for a genius; and an immense, possibly prohibitive amount will be required by the dull.

While acceleration models, in principle, hold the dependent variable, achievement, at a constant "mastery" level and allow time to "substitute" for aptitude, enrichment holds time constant and allows achievement to vary with aptitude (Walberg, 1971). Thus, as indicated in Figure 6c, achievement would be purely a function of aptitude if other production factors are held constant. If aptitude is normally distributed, achievement would be; and the achievement ranking of students after instruction would approximate the aptitude ranking before instruction.

The instructional theories of Carroll (1963) and Bloom (1976) may be interpreted as acceleration models within the production-theory framework. Caroll's formulation is

$$\text{Degree of learning} = f\left(\frac{\text{time actually spent}}{\text{time needed}}\right)$$

where "the numerator of this fraction will be equal to the smallest of the following three qualities: 1) opportunity--the time allowed for learning, 2) perseverance--the amount of time the learner is willing to engage actively in learning, and 3)--the amount of time needed to learn, increased by whatever amount necessary in view of the poor quality of instruction and lack of ability to understand less than optimal instruction. This last quantity (time needed to learn after adjustment for quality of instruction and ability to understand instruction) is also the denominator of the fraction" (Carroll, 1963, p. 730). Recast in the production function: 1) opportunity becomes quantity of instruction, that is, time allowed including self-instructional time; 2) perserverance becomes motivation (although an observer rating the percentage of engaged time is a reasonable proxy or even a more direct though expensive measure); 3) time needed becomes unadjusted ability; and 4) quality of instruction explicitly enters the equation rather than adjusting ability. Thus, rather than redefining all the independent variables in terms of time, the production function would assess the direct effect of each measured factor (including student age, the home environment, and the social environment of the class because of their known connections with achievement) in an explicit equation form.

Bloom's (1976) adaption of the Carrol model may be written

$$\begin{pmatrix} \text{Level of Achievement;} \\ \text{Affective Outcomes,} \\ \text{Rate of Learning} \end{pmatrix} = F \begin{pmatrix} \text{Quality of Instruction,} \\ \text{Cognitive Entry Behaviors,} \\ \text{Affective Entry Behaviors} \end{pmatrix}$$

In the production function, affective outcomes would be considered a particular type of achievement possibly involved in a feedback loop with motivation for subsequent achievement. Instead of measuring time to reach a criterion or gain in achievement divided by time to obtain a rate, time would directly enter the equation as an independent variable, quantity of instruction. The other variables have obvious correspondences in the production function. Thus, the sets of redefined variables in the Carroll and Bloom theories may be hypothesized to show substitutability and diminishing returns of the function.

In connection with time, a methodological point can be made. Bloom estimates that the slowest 10 percent of students may need 5-6 times as much time to learn as the most rapid 10 percent. Rarely are such large ratios encountered for variables in educational research. In a typical class, for example, the highest and lowest IQs might be 120 and 80, and the extreme scores on a 50-item achievement test might be 45 and 30; both ratios are 1.5 to 1. As emphasized in the earlier section on growth, educational measurements are severely limited in range and reliability. Even so, when achievement is related to time, the regression most often turns out to be logrithmic both in studies of school attendance, hours per day, and days per year (Karweit, 1976) and computer-assisted instruction (Suppes, 1972; Atkinson, 1972). Similarly in swimming, Arthur (1977) shows that at "1,000 yards a day one reaches 75% of the maximum training effort, and at 2,000 yards , 85%. In order to reach 95% of maximum, one needs to go 10,000 yards a day in training, and to get

99% to 100% one would need to go 16,000 yards a day" (p.2) which would take perhaps four hours. (My calculation of the variance accounted for by a logrithmic regression is .96 in contrast to .87 for the linear equation; nearly all the variance might be accounted for by bringing other production factors into the equation.) Although such things as ballet, music, chess, science, and writing performance cannot be measured so precisely, they appear to show diminishing returns relative to time: good or even excellent performance by the usual standards may only require a half to an hour a day of concentrated effort; but national rankings or one's best may require, among other things; three to ten hours of instruction and practice (pushing past an uncertain point, of course, may produce not only diminishing but negative returns).

Diminishing marginal returns to time can be well fitted by logrithmic transforms and can only be crudely approximated in the usual linear regression if the variation or reliability or both are minimal. On the other hand, to make a good estimate of the effect of any production factor such as time would require reasonably wide and reliable variation of the factor as well as achievement; moreover, interpolation within such data can be made more confidently than extrapolating beyond the range of narrow, colinear factors. A rival hypothesis to diminishing achievement returns, however, that requires investigation with wide-ranging tests, is that more and more students peak out on the conventional test's low ceiling as time goes by; that is, the test is incapable of detecting true linear returns at the upper range.

Aptitude-Treatment Interaction

The production function assumes that the factors interact by substituting for one another with diminishing returns. This trade-off differs from the way

that interactions are usually conceived by educational researchers, which is: students of type A achieve more using instructional method A, and students of type B achieve more using method B, as discussed in an earlier section. Among the problems of such usual conception are: 1) the plethora of constructs and measures of aptitudes and characterizations of instructional treatment that are used in a multitude of combinations by various investigators; 2) the usual reduction of complexes of continuous variables to a few dichotomies thereby eliminating much of precious reliable information in the data; 3) the obscure interpretations that are required of such interactions; 4) the extra degrees of freedom for estimation; and 5) the conclusions that such interactions apparently turn up no more often than chance (Bracht, 1970) and are rarely replicated (Cronbach and Snow, 1977; Walberg, 1976). Moreover, as noted earlier, they are scientifically unparsimonious and would be difficult to implement in ordinary classrooms.

In contrast, the production function approach: 1) narrows the selection of constructs and measures to those seven with a causally-plausible and empirically-verified ("main-effect") connection with achievement (the references cited above describe operational measures). 2) The continuous variation of the measure of each construct is brought into the equation (although, for more specific clues to the best improvements that might be made, it will be useful to disaggregate the parts of constructs such as quality of instruction and the classroom environment in some equations in addition to using overall indexes). 3) The interpretation of effect coefficients and policy forecasts are direct and straight forward; for example, increasing the instructional time or quality one percent increases achievement .2 percent. 4) The seven factors and the constant require eight degrees of freedom for the coefficients estimated in contrast to 28 for the conventional approach with main effects and first-order

interactions. 5) The correlations of achievement with the production factors replicate to varying degrees--ability, class, and home environment, and age most strongly and stably; motivation, quality and quantity of instruction, less so. But rarely have more than two or three been simultaneously analyzed, and never in production form. Thus, given the possible advantages of the production function over the older method of analyzing interactions, and the required presence of all causal variables in analyses that seek causal inference from non-randomly manipulated factors, it seems worthwhile trying to estimate the production coefficients in concert and probe the validity of the functional form.

On the other hand, the original approach to aptitude-treatment interactions requires more research (Cronbach and Snow, 1977). In these cases, the aptitude, quality, and their interactions would be brought into the production function as linear terms. The other production factors would then serve to increase the overall variance in achievement accounted for and therefore the precision of estimation of the interaction effects. Moreover, in the frequent case of non-randomized treatments, the presence of all the production factors in the regression will make causal inferences about the interactions more defensible.

Compensatory Education

During the 1960s, some policy makers and educational practitioners appeared to believe that increased instructional advantages can compensate for social and family disadvantages in the determination of achievement. Although this conception was somewhat vague, it often seemed to amount to the hypothesis of a linear trade-off between family and instructional environments. Such expectations resulted in frustration, because the returns in achievement to real increases in education expenditures and apparent improvements in schooling

seemed by no means linear; some even concluded that the marginal returns might be zero (see, for example, the references in the opening paragraph of this paper).

By the standards of the production function, adequate evidence and analyses of the returns are unavailable. Home environment was indicated by socioeconomic status, family size, ethnicity, objects in the home, and other sociological, anthropological, or psychological variables rather than the more educationally-relevent amount and intensity of adult-child interaction that account for several times as much variance in achievement (Walberg and Marjoribanks, 1976). In addition, the review of Rosenshine (1978) (1978) and others on productive factors in teaching that indicate predictively valid constructs and measures of instruction had not been completed. Because of poor measurement and the lack of evidence on the other production factors, the returns to compensatory programs are very poorly estimated.

Moreover, the returns to quality, although expected to be positive, are expected to be diminishing as shown in Figure 7a rather than linear. Aside from the a priori postulate of the production function that calls for such returns, evidence on the achievement "fan-spread" of individuals and "increasing gap" of groups over time supports the hypothesized relation. Campbell and Cook (1976) show that in educational programs for the disadvantaged, the relatively rich often get richer (Figure 7b); that is, the higher the student on a pre test, the higher not only his posttest score but his absolute and relative gain, even though the program is designed for students scoring lower on pre tests. Similarly, the Sesame Street television series confered a greater achievement benefit to advantaged groups than those for whom it was intended and thereby increased rather than reduced the gap (Cook and Campbell, 1976). The production function predicts

precisely such phenomena: the higher any factor becomes for all students, the smaller are the marginal returns (say, at about point n in Figure 15), and the greater the weight and relative determination of achievement by the other factors. Conversely, the lower the relative level (say, about point m in Figure 1) of a production factor, the greater are its marginal returns, and the more promising it would be to change, provided that budgets and other social and political forces allow it.

The fact that measures of home environment account for about 60 percent of the variance in achievement suggests that many students (and not just lower-income or minority ones) would achieve considerally more if the amount and intensity of intellectual stimulation in their home environments were increased. Indeed, the two field studies of school programs that were intended to help parents in highly disadvantaged neighborhoods to stimulate their children at home show that achievement gains can be doubled over comparable control students at least over 6 to 12 month periods. (Smith, 1958; Walberg, Bole, and Hirsch, 1977). The two programs cost very little and were met with enthusiasm by parents. Further investigation of such programs over longer periods using the production approach may bear out the promise of the initial field studies.

A Caveat

The educational production-function theory appears to explain some important phenomena concerning the relation of test achievement to production factors. The best probe or validation of the function would require longitudinal data on all seven factors and a wide-range achievement test that reflects the curriculum content. Data at only one or two time points, on only some of the factors, or on proxy or somewhat unreliable rather than optimal measures may prove valuable in partially testing the function.

Should the production function survive such tests, replicated values of the coefficients might be used to simulate and forecast the effects of changes. Given cost estimates and acceptability of such changes, the best strategy for reaching achievement goals could be chosen.

Even so, production economics also offers caution in attempting to maximize only one output. Despite the numerous ability and achievement tests available, they tend to measure only one major factor, verbal-educational accomplishment, surely an important aspect of human quality but by no means the only one. Performance on such tests predicts subsequent grades and performance on similar tests. For samples of adults that have attained the same amount of education, however, neither test scores nor grades predict indicators of success in later life such as income, participation in community activities, self-concept, supervisor- and peer-related effectiveness, and number of prizes, written works, patents, and other accomplishments. Rather, engagement in independent, self-sustained ventures and extracurricular activities during the high school and college years forecasts adult success (see reviews by Erickson, 1977, Jennings and Nathan, 1977, and Walberg, 1976).

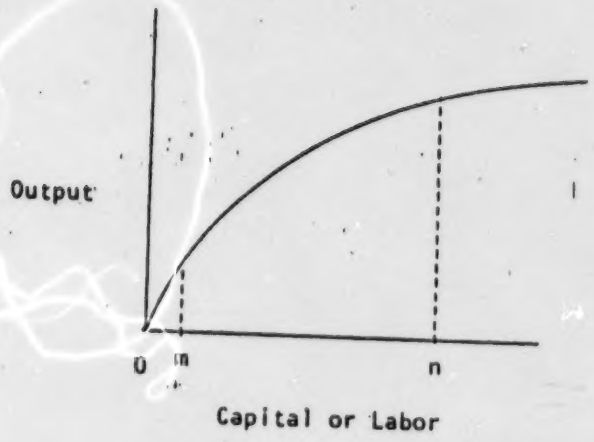
How much effort should be given to the production of test achievement? The "law of increasing relative costs" in economics states that to produce equal additional amounts of one good, other things being equal, larger and larger amounts of other goods must be sacrificed (Samuelson, 1970, p. 26). Thus in setting achievement goals on the usual tests and the earliest or most effective, cost-efficient, or rapid means of attaining them, educators, parents, and students should consider the extent to which other goals, opportunities, and experiences, valuable for the present or future, must be sacrificed. The causal mechanisms and the costs and benefits of other goals and their trade-offs with test achievement cannot be accurately

estimated. However, measures of student perceptions of the social environment of their classroom experience in terms of its difficulty, cohesiveness, democracy, goal direction, and other socio-psychological properties not only provide a demonstrably-valid indicator of a production factor for achievement but also possible educational process goals in their own right as a balance against test scores alone (Walberg, 1976). Additional productivity research on these variables is obviously in order and might contribute much to educational wisdom.

Even given the caveat concerning the diversity of educational goals and the limitations of outcome measures, economics and psychology have much to gain in joining forces to improve educational productivity. Psychologists have contributed much to establishing the consistent, strong correlates of educational achievement but have tended to examine the relation of achievement to only one or two production factors at a time in isolation from the others. Economists provide comprehensive frameworks for the study and improvement of productivity but in applying production models to education have accepted weak proxies such as student social class, school racial composition, and teacher salaries for the more direct and potent determinants of outcomes such as aptitude, quality of instruction and home environment. Combining the best of both disciplines is likely to be most fruitful.

Figure 1

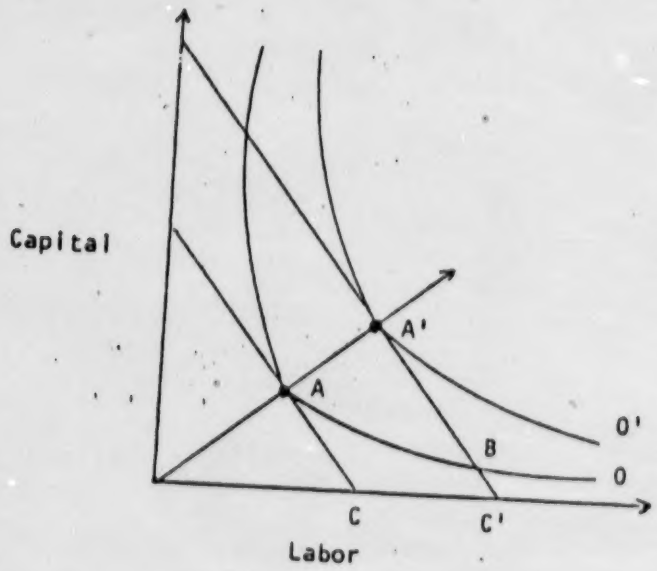
Diminishing Marginal Returns



Note: The effect of increasing one factor of production, captial or labor, while holding the other fixed is diminishing marginal returns, that is, smaller and smaller increases in output are produced by equal increases in the factor (say, around point n). At lower levels of either factor (point m), there may be at first increasing, then constant returns; at the highest, there may be negative returns, for example, too many cooks spoil the broth.

B 29
Figure 2

Equal-Product and Equal-Cost Lines



Note: Line O represents the various combinations of labor and capital required to produce a given, equal amount of output. Line C represents the combinations of labor and capital of a given costs. A is the point of lowest cost for a given output O since any other point, for example, B, requires higher costs. A' indicates the minimum cost for a higher quantity of output O' brought about by increasing capital and labor by the same amount, assuming their costs are equal for purposes of illustration, rather than either one excessively.

Figure 3

Natural Growth and Actual Output

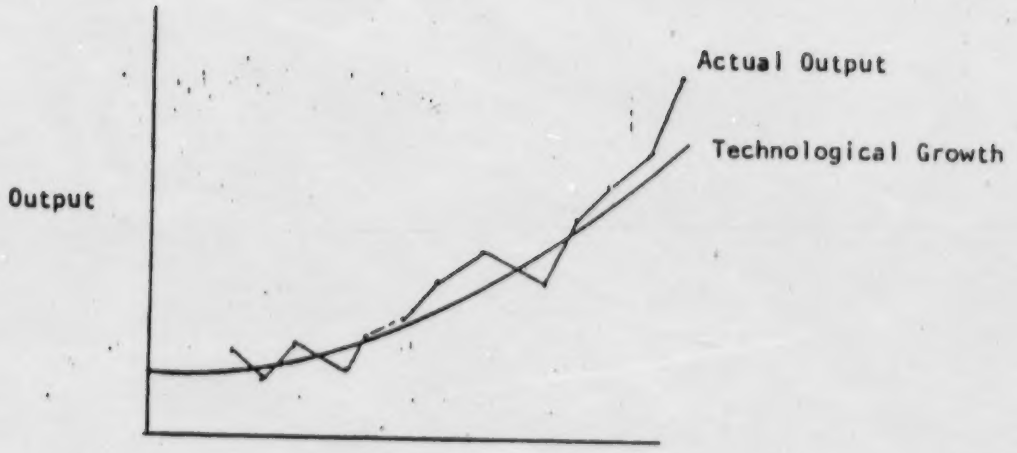
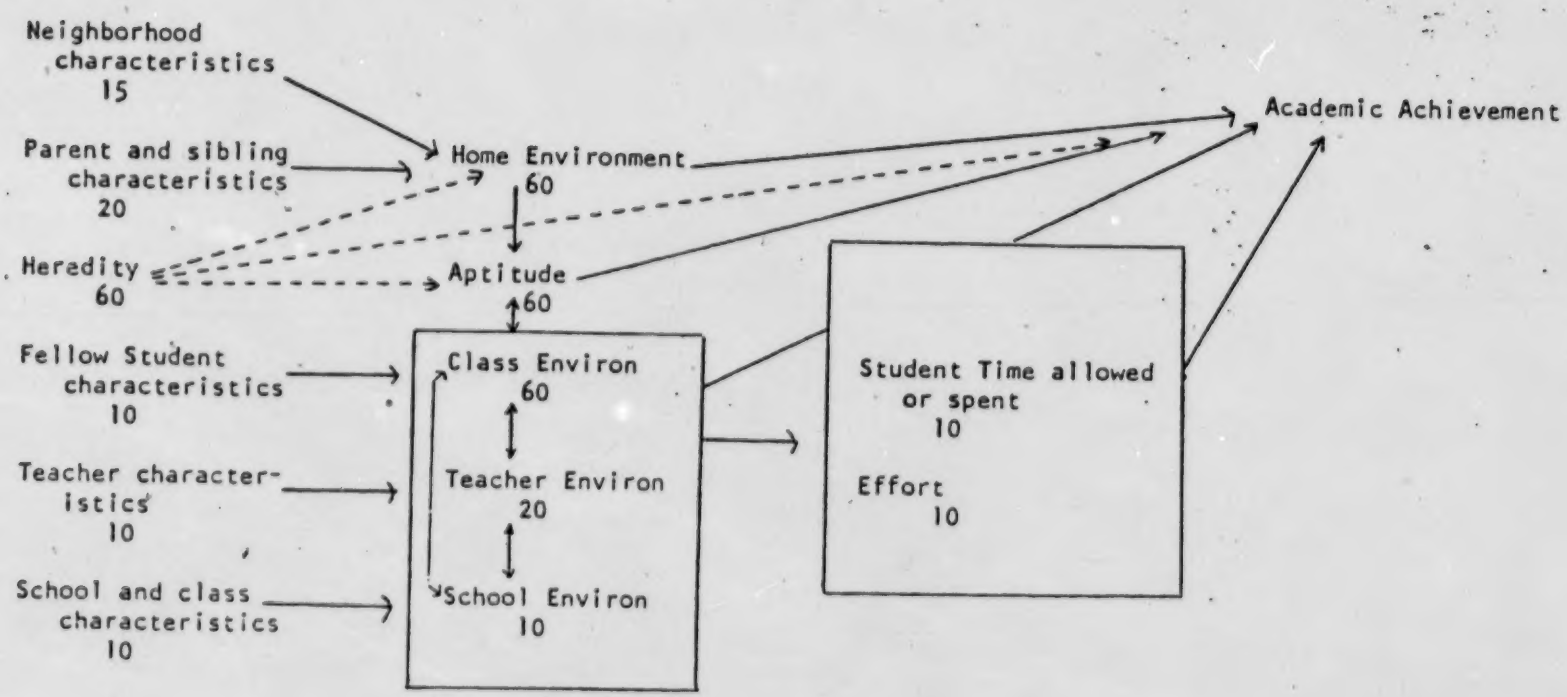


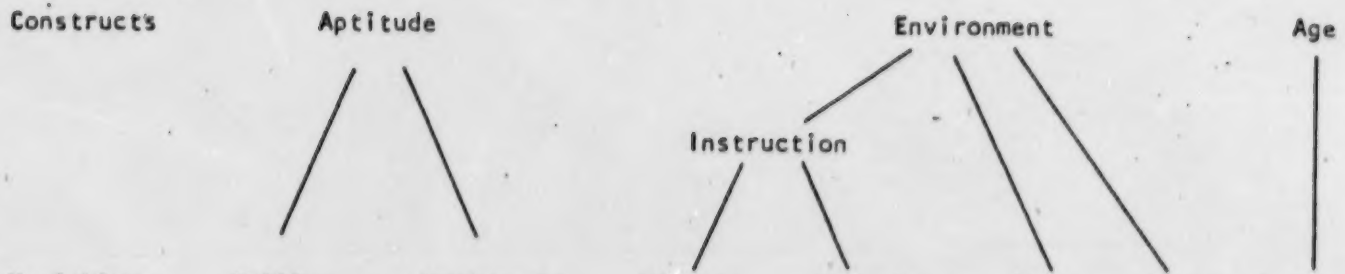
Figure 4:

Production Factors in School Learning



Note: The figures for each factor are representative estimates of the percentage of accounted-for variance in learning.

Figure 5



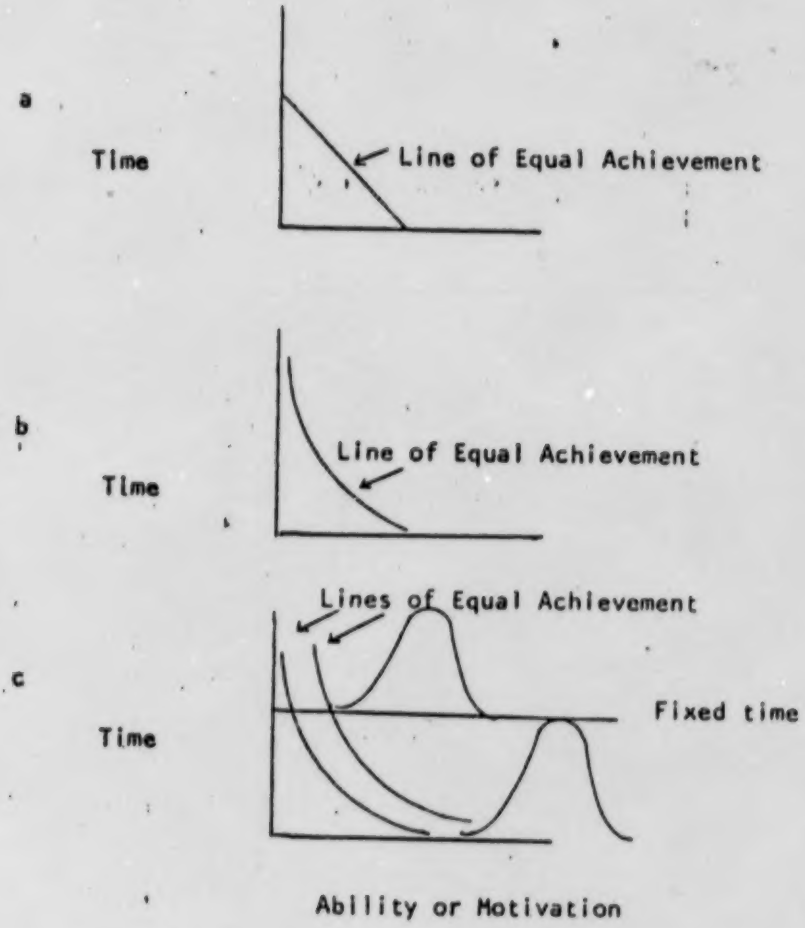
Reliability	.85	.75	.60	.60	.80	.80	.90
Achievement variance	'60	10	15	15	60	60	.80
Cost	5	10	30	30	10	100	2

Note: The reliability of each factor, the variance in achievement it accounts for, and the dollar cost per student to obtain the measurement are very rough estimates.

B 32

Figure 6

Acceleration and Enrichment as Functions of Aptitude and Time



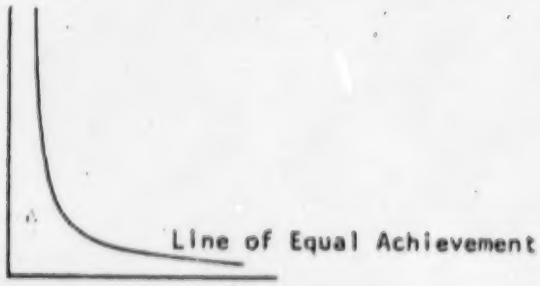
Note: The trade-off lines indicate equal amounts of achievement at each point on the line with compensating amounts of aptitude and time.

Figure 7

Problems of Compensatory Education

a

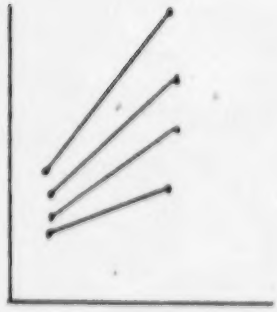
Home Environment



Quality of Instruction

b

Achievement



c

Achievement

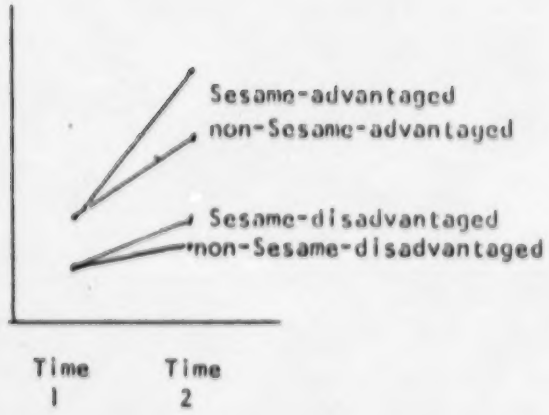
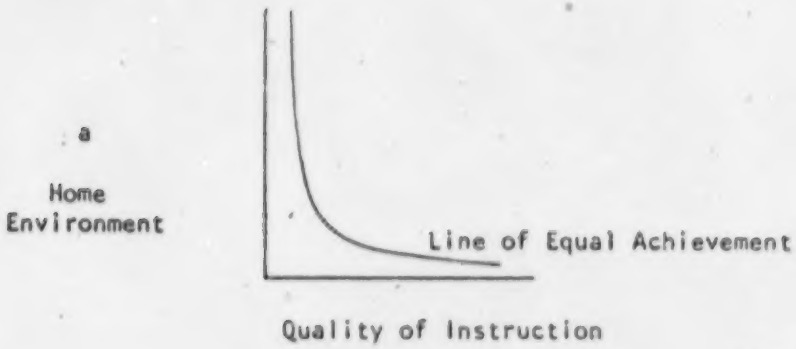


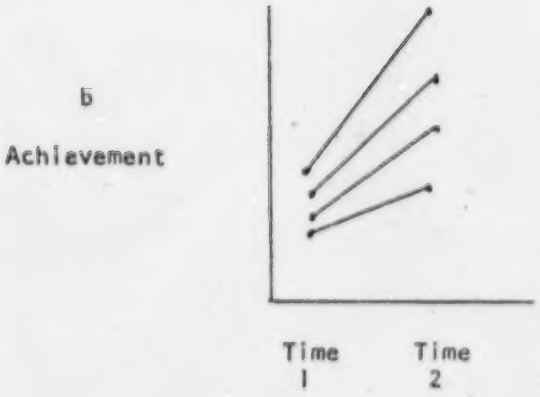
Figure 8

Problems of Compensatory Education

a



b



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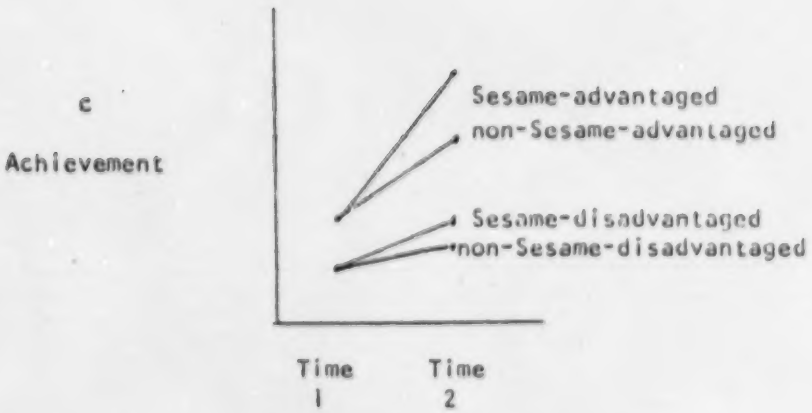

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